Chapter 5

Statistical and Systematic Errors for E989

E989 must obtain twenty-one times the amount of data collected for E821. Using the T method (see Section 16.1.2) to evaluate the uncertainty, 1.5×10^{11} events are required in the final fitted histogram to realize a 100 ppb statistical uncertainty. The systematic errors on the anomalous precession frequency ω_a, and on the magnetic field normalized to the proton Larmor frequency ω_p, are each targeted to reach the ± 70 ppb level, representing a threefold and twofold improvement, respectively, compared to E821. E989 will have three main categories of uncertainties:

- **Statistical.** The least-squares or maximum likelihood fits to the histograms describing decay electron events vs. time in the fill will determine ω_a, the anomalous precession frequency. The uncertainty $\delta\omega_a$ from the fits will be purely statistical (assuming a good fit). A discussion of the fitting sensitivity using various weighting schemes is given in Chapter 16, Section 16.2. The final uncertainty depends on the size of the data set used in the fit, which in turn depends on the data accumulation *rate* and the *running time*. These topics are discussed here.

- ω_a **Systematics.** Additional systematic uncertainties that will affect $\delta\omega_a$ might be anything that can cause the extracted value of ω_a from the fit to differ from the true value, beyond statistical fluctuations. Categories of concern include the detection system (e.g., gain stability and pileup immunity discussed in Chapter 16), the incoming beamline (lost muons, spin tracking), and the stored beam (coherent betatron oscillations, differential decay, E and pitch correction uncertainties). These latter topics are discussed in Chapter 4.

- ω_p **Systematics.** The magnetic field is determined from proton NMR in a procedure described in Chapter 15. The uncertainties are related to how well known are the individual steps from absolute calibration to the many stages of relative calibration and time-dependent monitoring. The"statistical" component to these measurements is negligible.

The purpose of this chapter is twofold. First, we summarize the event-rate calculation from initial proton flux to fitted events in the final histograms in order to determine the running time required to meet the statistical goals of the experiment. We also gather the results of many systematic uncertainty discussions that are described in various chapters throughout this document and roll up the expected systematic uncertainty tables for E989.

5.1 Event Rate Calculation Methodologies

The E989 Proposal [1] event-rate estimate was made by taking a **relative comparison approach** using like terms with respect to the known situation for rates in the E821 BNL experiment. Many factors allowed for trivial adjustments (proton fills per second, kinematics of the decay line length, kinematics of the decay line capture), while others relied on expected improvements in specific hardware components (optimized storage ring kicker pulse shape and magnitude, open-ended inflector, thinner or displaced Q1 outer plate and standoffs). In E821, the transmission through the closed-ended inflector and subsequently through the Q1 outer plates, followed by an imperfect kick, combined to give a sub-optimal storage ring efficiency factor, but individually the contributions from each element were not known as well as their product.

The E989 Conceptual Design Report [2] used that approach to estimate the need for a run duration of 17 ± 5 months, which included 2 months of overall commissioning and 2 months of systematic studies. The CDR also provided a bottom-up estimate, although at the time of the document, key simulations were just beginning. That approach suggested 18 months, perfectly in agreement with the relative calculation. Here, we present our estimate based on full **End-to-End Simulation** of the data accumulation rate. Many technical improvements since the CDR have tended to increase the overall data rate. However, the default use of the existing E821 inflector eliminates an anticipated gain. We have increased considerably from 2 to 6 the number of months that will be required to commission the entire accelerator chain and experiment.

5.1.1 Bottom-Up Event Rate Calculation

Table 5.1 contains a sequential list of factors that affect the event rate based on a bottom-up, full simulation approach. We assume the Proton Improvement Plan delivery of 4 batches of 4×10^{12} protons to the Recycler per 1.33 s supercycle with the Booster operating at 15 Hz. Each proton batch is split into four proton bunches of intensity 10^{12}; thus, the experiment will receive 16 proton bunches per supercycle, or a rate of 12 Hz. Each bunch corresponds to a "fill" of the Storage Ring. Four sequential stages of the simulation result in the estimates of positrons recorded by detectors per fill, and thus provide an estimate of the required operation of the experiment to achieve the statistical precision of 100 ppb stated in the Proposal and, importantly, the instantaneous rates on the many detector systems used in the experiment. The major simulation stages are:

1. Pion production on the target
2. Muon capture from pion decay, and subsequent transport to the storage ring entrance
3. Muon transmission into, and subsequent capture in, the storage ring

4. Muon decay positron acceptance by the detectors

The tools used include `MARS` for particle production, `G4beamline` and `BMAD` for beam transport and optimization, and `g2ringsim`, which is a `GEANT-4`-based full description of the storage ring and detector systems built in the `ART` framework. They are described in expert respective Chapters that follow. Here we present a linear narrative that will guide the reading of Table 5.1. The Table is then further justified with a sequence of Notes that pertain to each entry.

The particle flow is as follows. A burst of 10^{12} 8-GeV kinetic energy protons is focussed in the final stages of the M1 beamline to a spot size of 0.15 mm as it strikes the pion production target. The time distribution of the protons in the burst has an unusual "W-shaped" intensity profile with a maximum width of approximately 110 ns and a concentrated peak in the center. The target and lithium lens system was used previously for antiproton production. It is repurposed and optimized for the production and capture of 3.1 GeV/c positive particles in a fairly broad momentum bite. These are bent with the pulsed BMAG into the newly optimized M2 FODO lattice, which evolves following a short horizontal bend to the M3 beamline. The length of these sections, which is where the majority of muons are collected and captured, is approximately 270 m, where approximately 80% of the pions have decayed to muons.

The now mostly muon beam enters the Delivery Ring (DR) where it will make a variable number of revolutions (we anticipate 3 - 5) before being extracted by a fast kicker to the M5 beamline which delivers the beam to the $g-2$ Storage Ring entrance. The combination of beamlines so assembled admits at least a 40π mm-mrad phase space and has a momentum width $\delta p/p \sim 2\%$. The muon distribution retains the time profile described above. The purpose of this nearly 2 km path is to allow essentially all pions to decay to muons and to allow a time separation between muons and protons in the DR such that the protons can be removed by a kicker safely out of time from the passing muon burst. Thus, an essentially pure muon beam arrives at the Storage Ring at the magic muon momentum of 3.094 GeV/c. We assume that after a period of up to 6 months of steady commissioning and optimization, one can achieve $> 90\%$ transmission to the ring.

These muons must enter the Storage Ring through a hole in the back leg of the magnet yoke. They next enter a superconducting inflector magnet whose purpose is to null the strong return field flux that passes through the steel; it cancels the 1.4 T storage ring field over a 1.7 m path. This device is non-trivial. It has a small aperture, and includes coils covering both ends that introduce multiple scattering. The residual (non-canceled) fringe field along the path from the outside of the yoke to the exit of the inflector bends the beam left and right, the effect being to further restrict the transmission fraction. The beam emerges into the Storage Ring volume at an angle that is corrected by a ~ 12 mrad transverse outward kick during the first quarter turn. The newly designed magnetic kicker field profile in both space and time affects the storage efficiency. To reduce the muon loss rate for "stored" muons, the quadrupole system is used to scrape the beam along fixed collimaters and then return it to center. The transverse stored beam profile is reduced at the cost of $\sim 13\%$ of the muon flux.

Once stored—typically defined as a muon that remains in the storage volume for at least 100 turns—the muon decays can be studied using standard GEANT-based tools. To enhance the statistics in the subsequent simulations, we start by modeling the stored distribution with a polarized "muon gas" where we can then study the decays and detector acceptance and response.

Combined, the above sequence nets a yield of nearly 1100 recorded positrons per fill, each having an energy above the nominal threshold cut of 1.86 GeV, which maximizes the experimental sensitivity figure of merit. The yield is not more than 1.1×10^{-9}/pot. Therefore, each stage of the simulation requires optimized tools and an interface to subsequent phases through intermediate files. We note that full spin tracking is included. The following notes

Table 5.1: Event rate calculation using a bottom-up approach.

Item	**Factor**	**Value per fill**	**Note**
Protons on target		10^{12} p	1
Positive pions captured in FODO, $\delta p/p = \pm 0.5\%$	1.2×10^{-4}	1.2×10^{8}	2
Muons captured and transmitted to SR, $\delta p/p = \pm 2\%$	0.67%	8.1×10^{5}	3
Transmission efficiency after commissioning	90%	7.3×10^{5}	4
Transmission and capture in SR	$(2.5 \pm 0.5)\%$	1.8×10^{4}	5
Stored muons after scraping	87%	1.6×10^{4}	6
Stored muons after 30 μs	63%	1.0×10^{4}	7
Accepted positrons above E = 1.86 GeV	10.7%	1.1×10^{3}	8
Fills to acquire 1.6×10^{11} events (100 ppb)		1.5×10^{8}	9
Days of good data accumulation	17 h/d	202 d	10
Beam-on commissioning days		150 d	11
Dedicated systematic studies days		50 d	12
Approximate running time		402 ± 80 d	13
Approximate total proton on target request		$(3.0 \pm 0.6) \times 10^{20}$	14

explain entries in Table 5.1:

1. We assume a 0.15 mm spot size at the final focus of the M1 line on the target and an average proton pulse flux of 10^{12} from the Recycler, after a 4-fold split of the injected batch from the Booster.

2. MARS calculation. Assumes the (improved) proton spot size on target of 0.15 mm, which increased the yield compared to the measured rates at 0.5 mm spot size. Assumes 40π-mm-mrad emittance. Measurement verifies yield of positive particles. Simulation shows that 45% are pions. The target yield is assumed to be optimized by adjustments of the geometry compared to that in the CDR. Combined optimizations increased yield by the factor 1.35 compared to the CDR.

3. This is a multistep, full G4beamline simulation including all elements from the beginning of the M2 FODO, the bend to M3, three revolutions of the Delivery Ring, and transport along M5 to the last quad prior to the Storage Ring. Spin tracking gives a muon polarization average of 95%. Pions are assumed to have decayed; protons are kicked away in the DR from their time-of-flight lag.

4. After commissioning period of up to 6 months, estimate a 90% transmission from M2 to the end of M5, including losses in the DR kickers and accumulated misalignments of magnets. This is an expert opinion based on experience with the antiproton complex.

5. BMAD and g2ringsim calculations starting with muons from the output of G4beamline, which are transported through the back leg of the magnet, through the inflector (multiple scattering included), into the ring. They are kicked with a 20 ns rise time, 20 ns fall time, 80 ns flat top magnetic field. Losses occur from the apertures, the fringe fields, the non-ideal kicker pulse width and the natural Storage Ring acceptance. Both simulations suggest a storage fraction of $\sim (2.5 \pm 0.5)\%$.

6. We take the simple geometrical ratio of $(4.2/4.5)^2 = 0.87$ to establish a 2 mm annulus, given a position uncertainty of the quads of 0.5 mm.

7. Factor $exp(-t/\tau_\mu)$ with $t = 30$ μs and $\tau_\mu = 64.4$ μs.

8. Monte Carlo acceptance of the 24 calorimeters of 10.7% for events with energy above 1.86 GeV and striking the front face of one of the 24 calorimeter stations. Estimate includes all losses owing the material (quads, kicker plates, vacuum chambers).

9. With T method analysis, resolution of calorimeters folded in, and the polarization of 0.95 from the simulation, the asymmetry is $A = 0.38$ and the number of required events in the fit is 1.6×10^{11} for a 100 ppb statistical uncertainty.

10. Assume uptime data collection of 17 hours per day obtained as follows. One 3-h duration trolley run per 2 days loses 1.5 h/d. Accelerator uptime average is estimated at 85% and experiment livetime (including any functional downtime) is 90%.

11. Estimate of time to commission the new experiment and machine operation sequence. This is based, in part, on past experience at BNL and FNAL.

12. Generous estimate of dedicated systematic studies throughout the full measurement period.

13. Net data taking estimate. The range of $\pm 20\%$ is based on uncertainty in the storage fraction. Other factors may increase the uncertainty range.

14. Total proton request for the delivered beam to the experiment.

5.2 ω_a systematic uncertainty summary

Our plan of data taking and hardware changes addresses the largest systematic uncertainties and aims to keep the total combined uncertainty below 70 ppb. Experience shows that many of the "known" systematic uncertainties can be addressed in advance and minimized, while other more subtle uncertainties appear only when the data is being analyzed. Because we have devised a method to take more complete and complementary data sets, we anticipate the availability of more tools to diagnose such mysteries should they arise. Table 5.2 summarizes this section.

Table 5.2: The largest systematic uncertainties for the final E821 ω_a analysis and proposed upgrade actions and projected future uncertainties for data analyzed using the T method. The relevant Chapters and Sections are given where specific topics are discussed in detail.

Category	E821 [ppb]	E989 Improvement Plans	Goal [ppb]	Chapter & Section
Gain changes	120	Better laser calibration		
		low-energy threshold	20	16.3.1
Pileup	80	Low-energy samples recorded		
		calorimeter segmentation	40	16.3.2
Lost muons	90	Better collimation in ring	20	13.10
CBO	70	Higher n value (frequency)		
		Better match of beamline to ring	< 30	13.9
E and pitch	50	Improved tracker		
		Precise storage ring simulations	30	4.4
Total	180	Quadrature sum	70	

5.3 ω_p systematic uncertainty summary

The magnetic field is mapped by use of NMR probes. A detailed discussion is found in Chapter 15. In Table 5.3 we provide a compact summary of the expected systematic uncertainties in E989 in comparison with the final achieved systematic uncertainties in E821. The main concepts of how the improvements will be made are indicated, but the reader is referred to the identified text sections for the details.

Table 5.3: Systematic uncertainties estimated for the magnetic field, ω_p, measurement. The final E821 values are given for reference, and the proposed upgrade actions are projected. Note, several items involve ongoing R&D, while others have dependencies on the uniformity of the final shimmed field, which cannot be known accurately at this time. The relevant Chapters and Sections are given where specific topics are discussed in detail.

Category	E821 [ppb]	Main E989 Improvement Plans	Goal [ppb]	Chapter
Absolute field calibration	50	Special 1.45 T calibration magnet with thermal enclosure; additional probes; better electronics	35	15.4.1
Trolley probe calibrations	90	Plunging probes that can cross calibrate off-central probes; better position accuracy by physical stops and/or optical survey; more frequent calibrations	30	15.4.1
Trolley measurements of B_0	50	Reduced position uncertainty by factor of 2; improved rail irregularities; stabilized magnet field during measurements*	30	15.3.1
Fixed probe interpolation	70	Better temperature stability of the magnet; more frequent trolley runs	30	15.3
Muon distribution	30	Additional probes at larger radii; improved field uniformity; improved muon tracking	10	15.3
Time-dependent external magnetic fields	–	Direct measurement of external fields; simulations of impact; active feedback	5	15.6
Others †	100	Improved trolley power supply; trolley probes extended to larger radii; reduced temperature effects on trolley; measure kicker field transients	30	15.7
Total systematic error on ω_p	170		70	15

*Improvements in many of these categories will also follow from a more uniformly shimmed main magnetic field.

†Collective smaller effects in E821 from higher multipoles, trolley temperature uncertainty and its power supply voltage response, and eddy currents from the kicker. See 15.7.

References

[1] R. M. Carey, K. R. Lynch, J. P. Miller, B. L. Roberts, W. M. Morse, Y. K. Semertzides, V. P. Druzhinin and B. I. Khazin *et al.*,"The New (g-2) Experiment: A proposal to measure the muon anomalous magnetic moment to ± 0.14 ppm precision," FERMILAB-PROPOSAL-0989.

[2] [E989 Collalboration] J. Grange, et al., /it Muon $g-2$ Conceptual Design Report gm2-docdb #934, May 21 (2013).